Inspiring learning, achieving together

Progression in Written Methods for Addition, Subtraction, Multiplication and Division

What follows shows the stages to be taken in building up to compact, efficient methods for addition, subtraction, multiplication and division. Our aim is that children use mental methods (possibly with jottings) when appropriate. For calculations that they determine they cannot do in their head, they will have a written method which they can use accurately and with confidence. Time must be taken building up to the most efficient written method for all operations to ensure complete understanding.

Progression in Written Methods for Addition

To use an empty number line as below the children need to be able to:

§ *partition numbers into tens and ones*
§ *add multiples of 10 to any two-digit number*
§ *partition single digit numbers in order to bridge through multiples of 10*

Using an empty number line

The empty number line helps to record steps taken on the way to calculating the total.
The smallest number to be added should be partitioned into tens and ones.
The steps often bridge through a multiple of 10.

Example:

48 + 36 = 84
30 + 6

To use the methods outlined below, the children need to be able to:

§ partition numbers into hundreds, tens and ones

§ recall addition pairs to 9 + 9

§ add multiples of 10 together or multiples of 100 (such as 60 + 70 or 600 + 700) using a related fact (6 + 7) and knowledge of place value

§ mentally add multiples of 100, 10 and 1 e.g. 800 + 130 + 12

Partitioning

When adding larger numbers, it becomes less efficient to count on so partitioning is used.

Partition into (hundreds) tens and ones, add to form partial sums and then recombine.

Partitioning all the numbers mirrors the standard column method where ones are placed under ones and tens under tens etc.

Examples:

Partitioned numbers are written under one another:

47 + 76 = 40 + 7
70 + 6
110 + 13 = 123

375 + 567 = 300 + 70 + 5
500 + 60 + 7
800 + 130 + 12 = 942

256 + 328 = 200 + 50 + 6
300 + 20 + 8
500 + 80 + 4
10

Expanded column method

The expanded method leads children to the more compact column method so that they understand the structure and efficiency of it.

The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Example:

Write the numbers in columns:

```
      47
 +    76
      13
     110
       123
```

When moving from partitioning to the expanded method, emphasis must be placed on adding the ones first.

The addition of the tens in the calculation 47 + 76 is described as ‘forty plus seventy equals one hundred and ten’, stressing the link to the related fact ‘four plus seven equals eleven’.

Column method

The method is then shortened and when the column total is a two-digit number, the tens (or hundreds) are carried over into the next column. Use the words ‘carry ten’ or ‘carry one hundred’, **not** ‘carry one’.

Example:

```
   366
 + 458
   824
    1 1
```

Once learned, this method is quick and reliable. Extend to numbers with any number of digits and decimals with up to 3 places.

Progression in Written Methods for Subtraction

To use an empty number line as below the children need to be able to:

dentify the next multiple of 10 / 100 and use their knowledge of number facts to bridge to the next multiple of 10 / 100

Count on in multiples of 10 and / or 100

Using an empty number line

An empty number line should be used to find the difference.

The children should be taught to count on and bridge through multiples of 10 / 100.

Examples:

74 – 27 = 47

326 – 178 = 148

Counting back on an empty number line should only be used when the number to be subtracted is significantly smaller than the number being subtracted from. Children should be encouraged to count back mentally (using their knowledge of number facts / counting back in 10s) and when needed, an empty number line should serve as a quick jotting to aid mental calculation. Note the jump is drawn under the number line.

Examples:

23 – 4 = 19

19 20 23

-1 - 3

136 – 14 = 122

122 126 136

-4 - 10

Expanded column method

2 digit numbers with 1 adjustment needed: 74 – 27 =

Partition into tens and ones. Then say, ‘There is not enough to subtract 7 from 4,’ rather than, ‘You ***can’t*** subtract 7 from 4’. 70 + 4 is then adjusted to become 60 + 14. The calculation can now be carried out:

$$\overset{60}{70} + \overset{14}{4}$$

Over time, recording is refined to the formal written method below

2 digit numbers with1 adjustment needed: 74 – 27 =

$$\begin{array}{r} {}^{6}7\,{}^{1}4 \\ -\underline{2\ \ 7} \end{array}$$

− 20 + 7

40 + 7

3 digit numbers with1 adjustment needed: 563 – 271 =

Partition into hundreds, tens and ones. 500 + 60 needs to be adjusted to become 400 + 160. The calculation can now be carried out:

$$\begin{array}{r} \scriptstyle 400 \quad \scriptstyle 160 \\ 500 + 60 + 3 \\ -\ \underline{200 + 70 + 1} \\ 200 + 90 + 2 \end{array}$$

3 digit numbers with 2 adjustments needed: 563 – 278 =

This occurs when the tens ***and*** the ones to be subtracted are larger than those you are subtracting from. Firstly, 60+3 is adjusted to become 50+13. 500+50 is then adjusted to become 400+150. The calculation can now be carried out:

$$\begin{array}{r} \scriptstyle 150 \\ \scriptstyle 400 \quad \scriptstyle 50 \quad \scriptstyle 13 \\ 500 + 60 + 3 \\ -\ \underline{200 + 70 + 8} \\ 200 + 80 + 5 \end{array}$$

3 digit numbers with zeros where 2 adjustments are needed: 503 – 278 =

When 0’s are involved, the adjustments need to be done in a different order. There is not enough to subtract 8 from 3. As there are no tens, 500 + 0 is adjusted first to become 400 + 100. Then 100 + 3 can be adjusted to 90 + 13. The calculation can now be carried out:

90 13

4 7

Say ‘60 – 20’ or ‘6 tens – 2 tens’ not ‘6 – 4’

3 digit numbers with1 adjustment needed: 563 – 271 =

$$\begin{array}{r} ^{4}5\,^{1}6\ 3 \\ -\ \underline{2\ 7\ 1} \\ 2\ 9\ 2 \end{array}$$

Say ‘160 – 70’ or ‘16 tens – 7 tens’ not ‘16 – 7’

3 digit numbers with 2 adjustments needed: 563 – 278 =

$$\begin{array}{r} ^{4}5\,^{15}6\ ^{1}3 \\ -\ \underline{2\ 7\ 8} \\ 2\ 8\ 5 \end{array}$$

Say ‘150 – 70’ or ‘15 tens – 7 tens’ not ‘15 – 7’

3 digit numbers with zeros where 2 adjustments are needed: 503 – 278 =

$\begin{array}{r} \ \overset{400}{}\ \ \overset{100}{} \\ 500 + 0 + 3 \\ -\ \underline{200 + 70 + 8} \\ 200 + 20 + 5 \end{array}$	$\begin{array}{r} {}^{4}5\,{}^{91}0\,{}^{1}3 \\ -\ \underline{2\ 7\ 8} \\ 2\ 2\ 5 \end{array}$ Say '90 – 70' or '9 tens – 7 tens', not 9 – 7

Progression in Written Methods for Multiplication

Repeated addition

Children start by understanding multiplication as arrays and repeated addition. They use this understanding to help them work out multiplication facts they cannot recall quickly.

Example:

Looking at rows	Looking at columns
8 + 8 + 8 + 8 + 8	5 + 5 + 5 + 5 + 5 + 5 + 5 + 5
5 groups of 8	8 groups of 5
8 x 5	5 x 8

They show repeated addition on number line:

+8 +8 +8 +8 +8

0 8 16 24 32 40

Grid method

Children should move onto use grid method and use approximation to judge reasonableness of their answer.
The grid method allows the law of distribution to be applied, where the numbers are partitioned and part is multiplies separately. The products are then added to find the total product.

23 x 7 is approximately 20 x 10 = 200
Ensure that children understand the relationship between 7 x 2 and 7 x 20 and are not simply adding a zero.

72 x 38 is approximately 70 x 40 = 2800

Children to add together the numbers in each row first. Then, add together the total of each row, to find the final product.

372 x 24 is approximately 400 x 20 = 8000

Using a formal written method (taught according to the statutory requirements for children working at Year 4 mid onwards)

```
      38
×   _7
      56
  210
 266
```

Children describe what they are doing by referring to the value of the digits. Say, ‘30x7’ not ‘3x7’, although the relationship should be stressed.

```
      549
x    _6
      54
      240
  3000
  3294
```

Children say, ‘6x9, 6x40, 6x500’

```
      56
×     27
```

42
350
120
<u>1000</u>
1512

Children say, ‘7x6, 7x50, 20x6, 20x50’

37
x <u>${}_{4}6$</u>
222

Progression in Written Methods for Division

Repeated addition

Repeated addition allows the relationship between multiplication and division to be used.

Example without remainder:

40 ÷ 5   5 + 5 + 5 + 5 + 5 + 5 + 5 + 5

Ask, ‘How many 5s will fit into 40?’   = 8 fives

8 fives will fit into 40   40 ÷ 5 = 8

0   5   10   15   20   25   30   35   40

Example with a remainder:

38 ÷ 6

Ask, ‘How many 6s will fit into 38?’

6 + 6 + 6 + 6 + 6 + 6 + 2   = 6 sixes with a remainder of 2   6 sixes will fit into 38. There is a remainder of 2.

38 ÷ 6 = 6 r2

0 6 12 18 24 30 36 38

Chunking groups of the divisor on an empty number line

For larger numbers, when it becomes inefficient to count in single multiples, larger jumps can be recorded using known facts.

Example with a remainder:

41 ÷ 4 = 10 r1

(4 x 10)

0 40 41

Examples with quotients expressed as fractions or decimal fractions (only for the most able):

61 ÷ 4 = 15 ¼ or 15.25

(4 x 10) (4 x 5) + 1

676 ÷ 8 = 84.5

(8 x 80) (8 x 4) + 4

Children may simply record '10 groups' or '5 groups' within each jump to find the quotient.

Long division

72 ÷ 5 lies between 50 ¸ 5 = 10 and 100 ¸ 5 = 20

	72	
-	<u>50</u>	(10 groups - 5 x 10)
	22	
-	<u>20</u>	(4 groups - 5 x 4)

2
Answer : 14 r2

Children may simply record '10 groups' or '4 groups'.

256 ÷ 7 lies between 210 ¸ 7 = 30 and 280 ¸ 7 = 40

256
- 210 (30 groups - 7 x 30)
46
- 42 (6 groups - 7 x 6)
4
Answer: 36 r4

977 ÷ 36 is approximately 1000 ¸ 40 = 25

977
- 720 (20 groups - 36 x 20)
257
- 180 (5 groups - 36 x 5)
77
- 72 (2 groups - 36 x 2)
5
Answer: 27 r5

Short division (a.k.a. 'The Bus Stop Method')
Only to be taught as a method for dividing by a single-digit divisor. (- This is a statutory requirement from Year 5 onwards.)

Examples:
48 ÷ 4

1 2
4) 4 8

97 ÷ 4

2 4 r1
4) 9 7

627 ÷ 4

1 5 6 r3

4) $6_{2}2_{2}7$

724 ÷ 8

9 0 . 5

8) $7_{7}2$ 4 . $_{4}0$